Numeric Response Questions

Complex Numbers

Q.1 For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 41| = 5$, find the minimum value of $|z_1 - z_2|$.

Q.2 If the complex number z satisfies the condition $|z| \geq 3$, then find the least value of $|z + (1/z)|$.

Q.3 If $z = \left(\frac{1+i\sqrt{3}}{1+i}\right)^{25}$ and arg (z) $= \frac{25\pi}{k}$ then find k.

Q.4 The locus of $z = x + iy$ satisfying $\left|\frac{z-i}{z+i}\right| = 2$ is a circle then find radius of circle.

Q.5 If one of the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2 + \sqrt{3}i$ and centroid of the triangle formed by other vertices is $\left(k, \frac{-\sqrt{3}}{3}\right)$ then find k.

Q.6 If $\frac{2z_1}{3z_2}$ is a purely imaginary number, then $\left|\frac{z_1-z_2}{z_1+z_2}\right| =$

Q.7 If $(\sqrt{8} + \mathrm{i}^{50} = 3^{49}(\mathrm{a} + \mathrm{ib})$, then find value of $\mathrm{a}^2 + \mathrm{b}^2$,

Q.8 If $|z_1| = |z_2| = |z_1| = 1$ and $z_1 + z_2 + z_s = 0$, If the area of the triangle whose vertices are z_1, z_2, z_3 is $\sqrt{3}k$, then find k.

Q.9 If $|z_1| = 1, |z_2| = 2, |z_s| = 3$ and $|9z_{1z_2} + 4z_iz_1 + z_zz_1| = 12$, then the value of $|z_1 + z_2 + z_s|$ is equal to :

Q.10 If Re $\frac{(1+\mathrm{i})^2}{3-\mathrm{i}}$ is k then find [k] (where [.] denotes greatest integer function).

Q.11 If m & M denotes the minimum and maximum value of $|2z + 1|$ where $|z - 2i| \leq 1$, then find (m+M)?.

Q.12 If $\left|\frac{z_1-3z_2}{3-z_1z_2}\right| = 1$ and $|z_2| \neq 1$, then find $|z_1|$.

Q.13 If $x^2 - x + 1 = 0$, then find value of $\sum_{n=1}^{5}\left(x^n + \frac{1}{x^n}\right)^2$.

Q.14 If $z = x - iy$ and $z^{13} = p + iq$, then find value of $\frac{\left(\frac{x}{p}+\frac{y}{q}\right)}{(p^2+q^2)}$.

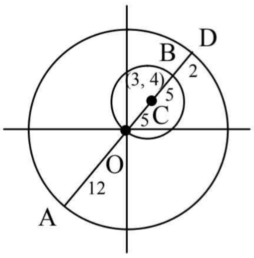
Q.15 If $z = \frac{\pi}{4}(1 + i)^4\left(\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right)$, then find $\left(\frac{|z|}{\text{Amp }(z)}\right)$.

ANSWER KEY

1. 2.00	**2.** 2.67	**3.** 12.00	**4.** 1.33	**5.** 0.67	**6.** 1.00	**7.** 9.00
8. 0.75	**9.** 2.00	**10.** –1.00	**11.** 68.00	**12.** 3.00	**13.** 8.00	**14.** –2.00
15. 4.00						

Hints & Solutions

1.



$|z_1 - z_2|_{min} = BD = 2$

2. $\left|z+\frac{1}{z}\right| \geq \left||z|-\frac{1}{|z|}\right|$

The least value of occurs when $|z| = 3$

$\left|z+\frac{1}{z}\right|_{min.} = 3 - \frac{1}{3} = \frac{8}{3}$

3. $z = \frac{(1+i\sqrt{3})^{25}}{(1+i)^{25}} = \frac{2^{25}.(e^{i\pi/3})^{25}}{2^{\frac{25}{2}}.(e^{i\pi/4})^{25}}$

$= 2^{\frac{25}{2}}.e^{i\left(\frac{25\pi}{12}\right)} = e^{\frac{25}{2}}.e^{i\left(\frac{\pi}{12}\right)}$

or $\left(2\pi+\frac{\pi}{12}\right) = \frac{25\pi}{12}$

4. $x^2 + (y - 1)^2 = 4\,[x^2 + (y + 1)^2]$

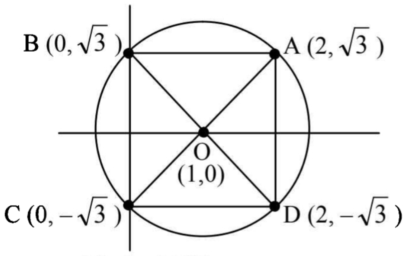
$3\,(x^2 + y^2) + 8y + 2y + 3 = 0$

$x^2 + y^2\ \frac{10}{3}\ y + 1 = 0$

radius $= \sqrt{\frac{25}{9}-1}$

$= \frac{4}{3}$

5.



Centroid of Δ BCD

$= \left(\frac{0+0+2}{3}, \frac{\sqrt{3}+(-\sqrt{3})+(-\sqrt{3})}{3}\right)\left(\frac{2}{3}, -\frac{\sqrt{3}}{3}\right)$

6. As given, let $\frac{2z_1}{3z_2} = iy$ or $\frac{z_1}{z_2} = \frac{3}{2}iy$, so that

$\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = \left|\frac{\frac{z_1}{z_2}-1}{\frac{z_1}{z_2}+1}\right| = \left|\frac{\frac{3}{2}iy-1}{\frac{3}{2}iy+1}\right| = \left|\frac{1-\frac{3}{2}iy}{1+\frac{3}{2}iy}\right| = 1$

$\{\because |z| = |\bar{z}|\}$

7. $(\sqrt{8}+i)^{50} = 3^{49}(a+ib)$

Taking modules and squaring both sides, we get

$(8 + 1)^{50} = 3^{98}\,(a^2 + b^2)$

$9^{50} = 3^{98}(a^2 + b^2)$

$3^{100} = 3^{98}\,(a^2 + b^2)$

$\Rightarrow (a^2 + b^2) = 9$

8. $|z_1| = |z_2| = |z_3| = 1 \Rightarrow$ circumcentre of triangle is origin. Also orthocentre $z_1 + z_2 + z_3 = 0$, which coincide with circumcentre, Then triangle is equilateral.

As radius is 1, length of side is $a = \sqrt{3}$.

Then area of triangle $\frac{\sqrt{3}}{4}a^2 = \frac{3\sqrt{3}}{4}$.

9. $|z_1| = 1 \Rightarrow z_1\bar{z}_1 = 1,\ |z_2| = 2 \Rightarrow z_2\bar{z}_2 = 4,$

$|z_3| = 3$

$\Rightarrow z_3\bar{z}_3 = 9$

Also, $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$

$\Rightarrow |z_1z_2z_3\bar{z}_3 + z_1z_2z_3\bar{z}_2 + z_1\bar{z}_1z_2z_3| = 12$

$\Rightarrow |z_1z_2z_3|\ |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 12$

$\Rightarrow |z_1|\ |z_2|\ |z_3|\ |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 12$

$\Rightarrow 6|\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 12$

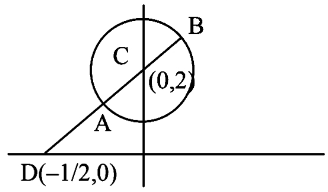
$\Rightarrow |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 2$

$\Rightarrow |z_1 + z_2 + z_3| = 2.$

10. $\text{Re}\left[\frac{(1+i)^2}{3-i}\right] = \text{Re}\left[\left(\frac{2i}{3-i}\right)\left(\frac{3+i}{3+i}\right)\right]$

$= \text{Re}\left[\frac{6i-2}{9+1}\right] = \text{Re}\left[-\frac{2}{10}+\frac{6}{10}i\right] = -\frac{1}{5}$

11. m = 2AD = 2(CD – AC)



$m = 2\left(\frac{\sqrt{17}}{2}-1\right) = \sqrt{17}-2$

M = 2BD = 2(CD + BC)

$M = 2\left(\frac{\sqrt{17}}{2}+1\right) = \sqrt{17}+2$

$\Rightarrow (m + M)^2 = 68$

12. $(z_1 - 3z_2)(\bar{z}_1 - 3\bar{z}_2) = (3 - z_1\bar{z}_2)(3 - \bar{z}_1 z_2)$

$\Rightarrow |z_1|^2 - |z_1|^2\,|z_2|^2 + 9|z_2|^2 - 9 = 0$

$\Rightarrow (1 - |z_2|^2)(|z_1|^2 - 9) = 0$

$\Rightarrow |z_1| = 3$

13. $x^2 - x + 1 = 0$

$\Rightarrow x = \frac{1 \pm \sqrt{3}i}{2} = -\omega, -\omega^2$

$\therefore \sum_{n=1}^{5}\left(x^{2n} + \frac{1}{x^{2n}} + 2\right)$

$\Rightarrow \left(x^2 + \frac{1}{x^2} + 2\right) + \left(x^4 + \frac{1}{x^4} + 2\right) +$

$\left(x^6 + \frac{1}{x^6} + 2\right) + \left(x^8 + \frac{1}{x^8} + 2\right) +$

$\left(x^{10} + \frac{1}{x^{10}} + 2\right)$

$\Rightarrow (\omega^2 + \omega^4 + \omega^6 + \omega^8 + \omega^{10}) +$

$\left(\frac{1}{\omega^2} + \frac{1}{\omega^4} + \frac{1}{\omega^6} + \frac{1}{\omega^8} + \frac{1}{\omega^{10}}\right) + 10$

$\Rightarrow -1 - 1 + 10 = 8.$

14. $x - iy = (p + iq)^3 = {}^3C_0p^3 + {}^3C_1p^2(iq) + {}^3C_2\, p(iq)^2 + {}^3C_3(iq)^3$

$x = p^3 - 3pq^2,\ -y = 3p^2q - q^3$

$\frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2 \Rightarrow \frac{\frac{x}{p}+\frac{y}{q}}{p^2+q^2} = -2$

15. $z = \frac{\pi}{4}((1 + i)^2)^2 \times$

$\left(\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i} \times \frac{\sqrt{\pi}-i}{\sqrt{\pi}-i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i} \times \frac{1-\sqrt{\pi}i}{1-\sqrt{\pi}i}\right)$

$z = -\pi(-i - i)$

$z = 2\pi i$

so, $|z| = 2\pi$ and $\text{Amp.}(z) = \frac{\pi}{2}$

then $\frac{|z|}{\text{Amp.}(z)} = \frac{2\pi}{\pi/2} = 4.$